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Research article

MATHEMATICAL MODEL TO PREDICT THE RATE OF SEEPAGE FORCE SUBJECTED TO VISCOUS FRICTION AND DRAG FORCE IN CLAY AND SILTY SOIL FORMATION

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Abstract

The developments of mathematical model to monitor the rate of seepage force in the soil are based on the behaviour of the formation in terms of fluid flow conditions. These are under the influences of geophysical properties of the soil. Subject to these conditions, the expression of viscous friction and drag forces in clay and silty formations were observed to develop heterogeneous seepage forces in deltaic formations, such condition developed some behaviuor that required mathematical model to express the behavioral influences from these soil properties to predict the rate of seepage forces in deltaic formation. When water flows through the soil, the water head is dissipated in viscous friction during energy dissipation, the drag forces is exerted on the soil particles in the direction of flow, it confirms that when water deposits in the soil column, the fluid height in the surface are raised, whereby the water pressured at the bottom of the soil sample increased and the drag forces of the soil particles become greater, the height of the buoyant weight of the particles are found to be in a balanced state at critical height, the expressed governing equations were developed under the influence of these conditions as it is stated to generate seepage forces in a deltaic formation. The expressed model developed will monitor and predict the rate of seepage forces in clay and silty formations.

Keywords: Mathematical models, seepage forces, viscous friction, drag forces, clay and silty formation

1. Introduction

Groundwater is frequently encountered in construction projects, the movement of water through soil is known to be seepage and such movement leads to several groups of problem in Civil Engineering. Seepage of water has a bearing on three major types of problems; the loss of stored water through an earth dams foundation (ii) instability of slopes and foundation of hydraulic structure due to the force exerted by the percolating water (iii) settlement of structure founded on or above compressibility layers due to explosion of water from void caused by lead applications. When water flows through soil, the water is dissipated in viscous friction. During energy dissipation, a drag force is exerted on the soil particles in the direction of flow. In most instant column of soil the height H of the water surface in the reservoir is raised, the water pressure at the bottom of the soil is increased and the drag force on the soil becomes greater. The drag force and the buoyant weight of the particles are in balance at a critical height $h = h_s$ and an increase in height will cause the soil particles to be washed out of the container. At this critical condition, the force acting on the bottom of the soil sample will just be equal to the weight of the soil and water mass of the container [21]. Further more in relation to seepage forces earth dams are formed by different zones constructed of especial material; furthermore, each zone has effect on stability and operation of earth dams. Because in zoned earth dams different particle with various size are settled beside each other, water infiltrate among them. This situation continues gradually until difference between water potential in up and down stream cause moving particles from porosity and starting piping suddenly. This event is named as seepage. [5,6] In theory, seepage is named gravitation water which flow into soil particles porosity in result of gravity. Seepage in dam is similar to flow in an open channel because their free surface and atmosphere pressure. But, underground water is including two sections liquid/solid; and, flow velocity and discharge are controlled by soil permeability. A method to display flow in body and foundation of earth dam is Flow net. The primary line in flow net is named as phreatic line which restricts seepage. In spite of existing different permeability in soils, when the ratio of horizontal to vertical permeability is same, the phreatic line will reach identical position eventually [4].

More so, centrifuge modelling technique is being used to study many different aspects of soil behaviour. Flow velocity in a centrifuge model at Ng will be N times faster compared to the prototype it represents. Consequently the scaling law for seepage velocity has been established as $m\ p\ v = N\ v$ [18] and has been confirmed experimentally [[8,9]. This scaling law for seepage velocity has been accepted and commonly used, but the question of whether it is the Darcy's permeability (hydraulic conductivity) or the hydraulic gradient that is a function of gravity has not been addressed properly. This issue was highlighted 14], who points out to the multiplicity of the concepts in scaling flow velocity. [10] And [12] also discussed this issue. [17,11] , [19] and more recently [18] are among many others who have considered permeability (k) to be directly proportional to gravity and hydraulic gradient (i) to be independent of gravity. While this explains why seepage velocity has a scaling law of N (m $p\ v = N\ v$), there is an alternative explanation for the increase of seepage velocity in a centrifuge. [18, 13,14,15,16, 19 and 20] have all suggested that permeability to be independent of gravity and it is the hydraulic gradient which has got a scaling factor of N. Since both sides of the explanation result in the same final answer $m\ p\ v = N\ v$ and it is the final seepage velocity that is considered important in many cases, the controversy has often been overlooked.

2. Governing equation

$$n_e \frac{\partial s_f}{\partial t} = \frac{Gy_w}{1+e} \frac{\partial {s_f}^2}{\partial z^2} - y_w \frac{\partial s_f}{\partial z} - AL \frac{\partial s_f}{\partial z} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

But for simplicity let $\frac{Gy_w}{1+e}$ and $\frac{y_w}{1+e}$ be denoted as ϕ and φ

Therefore, the equation can be written as:

$$n_e \frac{\partial s_f}{\partial t} = \phi \frac{\partial {s_f}^2}{\partial z^2} - \varphi \frac{\partial s_f}{\partial z} - AL \frac{\partial s_f}{\partial z} \qquad \ldots\ldots\ldots\ldots \qquad (2)$$

Let $s_f = ZT$ from equation (2), we have

$$n_e T^1 Z = \phi Z^{11} T - \varphi Z^1 T - ALZ^1 T \qquad \ldots\ldots\ldots\ldots \qquad (3)$$

$$n_e \frac{T^1}{T} = \phi \frac{Z^{11}}{Z} - \varphi \frac{Z^1}{Z} - AL \frac{Z^1}{Z} = \gamma^2 \qquad \ldots\ldots\ldots\ldots \qquad (4)$$

$$n_e \frac{T^1}{T} = \gamma^2 \qquad \ldots\ldots\ldots\ldots \qquad (5)$$

$$\phi \frac{Z^{11}}{Z} = \gamma^2 \qquad \ldots\ldots\ldots\ldots \qquad (6)$$

$$\varphi \frac{Z^1}{Z} = \gamma^2 \qquad \ldots\ldots\ldots\ldots \qquad (7)$$

$$AL \frac{Z^1}{Z} = \gamma^2 \qquad \ldots\ldots\ldots\ldots \qquad (8)$$

This implies that equation (6), (7) and (8) can be written as:

$$[\phi - \varphi - AL] \frac{Z}{Z} = \gamma^2 \qquad \ldots\ldots\ldots\ldots \qquad (9)$$

From (4)

$$n_e \frac{T^1}{T} = \gamma^2$$

i.e. $n_e \frac{dT}{dt} = \gamma^2$ $\qquad \ldots\ldots\ldots\ldots \qquad (10)$

$$\int \frac{dT}{dt} = \frac{\gamma^2}{n_e} \int dt \qquad \ldots\ldots\ldots\ldots \qquad (11)$$

$$Ln\, T = \frac{\gamma^2}{n_e} t + C_1 \qquad \ldots\ldots\ldots\ldots \qquad (12)$$

$$\frac{\gamma^2}{n_e} + C_1 \qquad \ldots\ldots\ldots\ldots \qquad (13)$$

$$T = A \ell^{\frac{\gamma^2}{n_e}} \qquad \ldots\ldots\ldots\ldots \qquad (14)$$

From (9)

$$[\phi - \varphi - AL]\frac{Z}{Z} = \gamma^2 dz \quad \ldots\ldots\ldots\ldots \quad (15)$$

$$\int \frac{dz}{dz} = \frac{\gamma^2}{\phi - \varphi - AL}\int dz \quad \ldots\ldots\ldots\ldots \quad (16)$$

$$Lnz = \frac{\gamma^2}{\phi - \varphi - AL} + C_1 \quad \ldots\ldots\ldots\ldots \quad (17)$$

$$z = \exp\left[\frac{\gamma^2}{\phi - \varphi - AL} z + C_1\right] \quad \ldots\ldots\ldots\ldots \quad (18)$$

$$\boxed{z = B\exp\frac{\gamma^2}{\phi - \varphi - AL}} \quad \ldots\ldots\ldots\ldots \quad (19)$$

Combining (18) and (19), we have

$$s_{f1}\ TZ = ZT$$

$$= A_e\, n_e\, B\left[\exp\frac{\gamma^2}{\phi - \varphi - AL}\right] \quad \ldots\ldots\ldots\ldots \quad (20)$$

$$s_f[z,T] = AB\exp\left[\frac{t}{n_e} + \frac{Z}{\phi - \varphi - AL}\right]\gamma^2 \quad \ldots\ldots\ldots\ldots \quad (21)$$

$$s_f[z,T] = AP\exp\left[\frac{\frac{d}{v}}{n_e} + \frac{V\cdot t}{\phi + \varphi - AL}\right]\gamma^2 \quad \ldots\ldots\ldots\ldots \quad (22)$$

3. Materials and Method

Standard laboratory experiment where performed to monitor the rate of flow under seepage condition at different formation, the soil deposition of the strata were collected in sequences base on the structural deposition at different locations, this samples collected at different location generate variation at different depth producing different migration of fluid flow developing seepage force at different strata, the experimental results are applied to compare with the theoretical values to determine the validation of the model.

4. Result and Discussion

Results and discussion are presented in tables including graphical representation of seepage force at different strata

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Table 1: Theoretical values of seepage force at different depths

Depth [m]	Theoretical Values of seepage force
2	5.611
4	5.623
6	5.635
8	5.646
10	5.658
12	5.67
14	5.682
16	5.694
18	5.706
20	5.717
22	5.729
24	5.741
26	5.753
28	5.765
30	5.779

Table 2: Theoretical values of seepage force at different Time

Time	Theoretical Values of seepage force
2	5.611
4	5.623
6	5.635
8	5.646
10	5.658
12	5.67
14	5.682
16	5.694
18	5.706
20	5.717
22	5.729
24	5.741
26	5.753
28	5.765
30	5.779

Table 3: Theoretical values of seepage force at different depths

Depth [m]	Theoretical Values of seepage force
3	5.6
6	5.71
9	5.77
12	5.83
15	5.89
18	5.95
21	6.01
24	6.07
27	6.13
30	6.19

Table 4: Theoretical values of seepage force at different depths

Time per day	Theoretical Values of seepage force
10	5.6
20	5.71
30	5.77
40	5.83
50	5.89
60	5.95
70	6.01
80	6.07
90	6.13
100	6.19

Table 5: Comparison of Theoretical and Measured Values of Seepage Force at Different Depth

Depth [m]	Theoretical Values of seepage force	Measured Values
2	5.611	5.608
4	5.623	5.618
6	5.635	6
8	5.646	5.738
10	5.658	5.748
12	5.67	5.758
14	5.682	5.768
16	5.694	5.778
18	5.706	5.788
20	5.717	5.798

22	5.729	5.808
24	5.741	5.818
26	5.753	5.828
28	5.765	5.838
30	5.779	5.848

Table 6: Comparison of Theoretical and Measured Values of Seepage Force at Different Time

Time	Theoretical Values of seepage force	Measured Values
2	5.611	5.615
4	5.623	5.626
6	5.635	5.639
8	5.646	5.648
10	5.658	5.653
12	5.67	5.668
14	5.682	5.684
16	5.694	5.698
18	5.706	5.701
20	5.717	5.714
22	5.729	5.724
24	5.741	5.738
26	5.753	5.755
28	5.765	5.761
30	5.779	5.775

Table 7: Comparison of Theoretical and Measured Values of Seepage Force at Different Depth

Depth [m]	Theoretical Values of seepage force	Measured Values
3	5.6	5.63
6	5.71	5.69
9	5.77	5.75
12	5.83	5.81
15	5.89	5.87
18	5.95	5.93
21	6.01	5.99
24	6.07	6.05
27	6.13	6.11
30	6.19	6.17

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Table 8: Comparison of Theoretical and Measured Values of Seepage Force at Different Depth

Time per day	Theoretical Values of seepage force	Measured Values
10	**5.6**	**5.59**
20	**5.71**	**5.69**
30	**5.77**	**5.75**
40	**5.83**	**5.79**
50	**5.89**	**5.84**
60	**5.95**	**5.91**
70	**6.01**	**5.99**
80	**6.07**	**6.04**
90	**6.13**	**6.11**
100	**6.19**	**6.15**

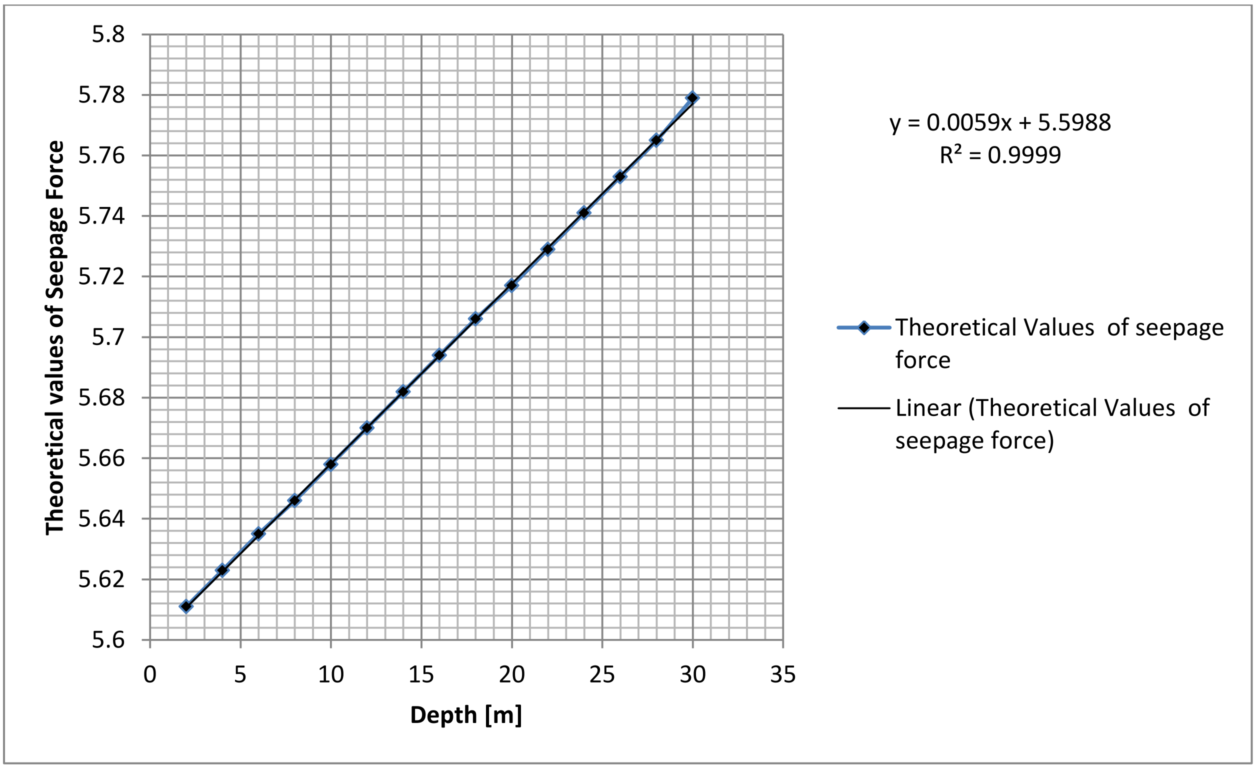

Figure 1: Theoretical values of seepage force at different depths

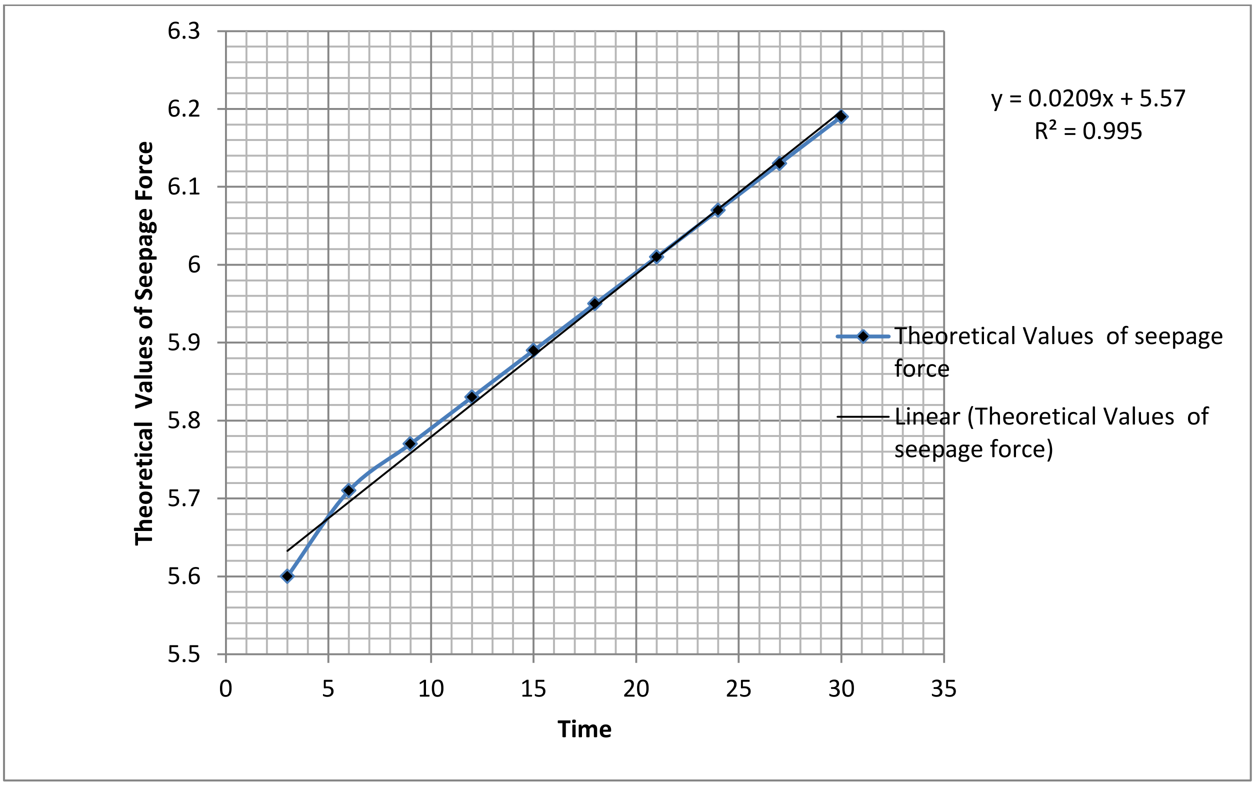

Figure 2: Theoretical values of seepage force at different Time

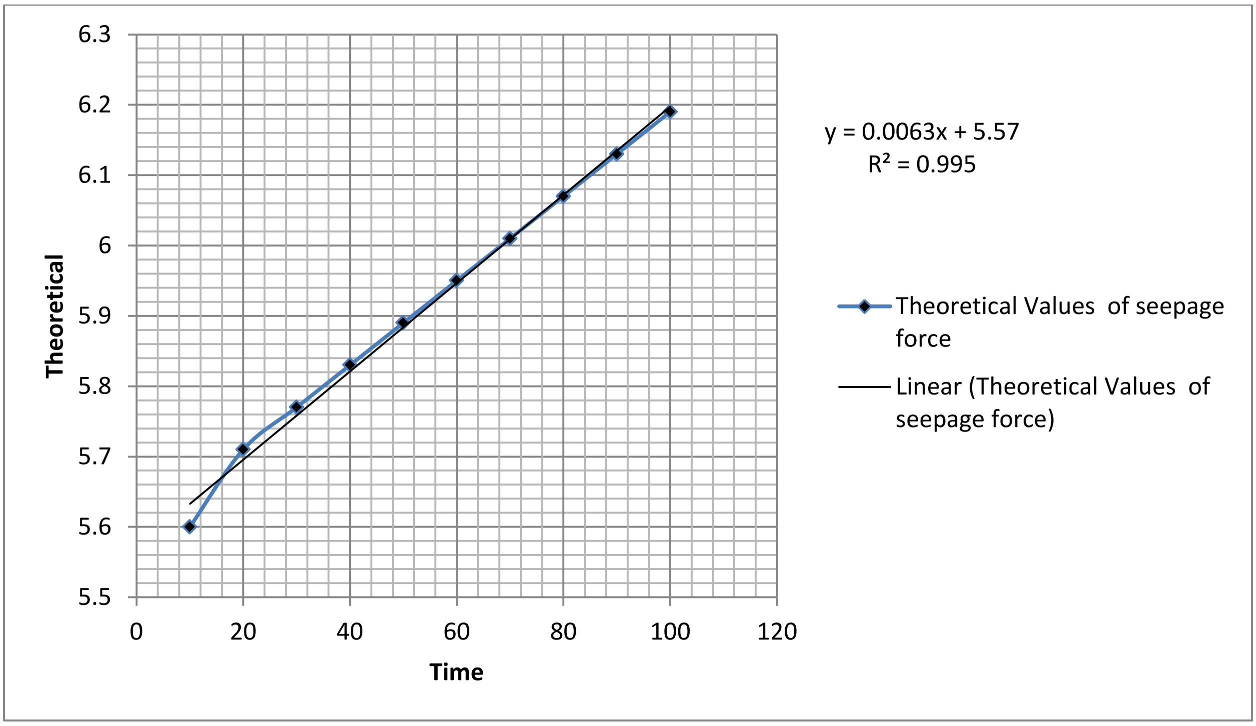

Figure 3: Theoretical values of seepage force at different Time

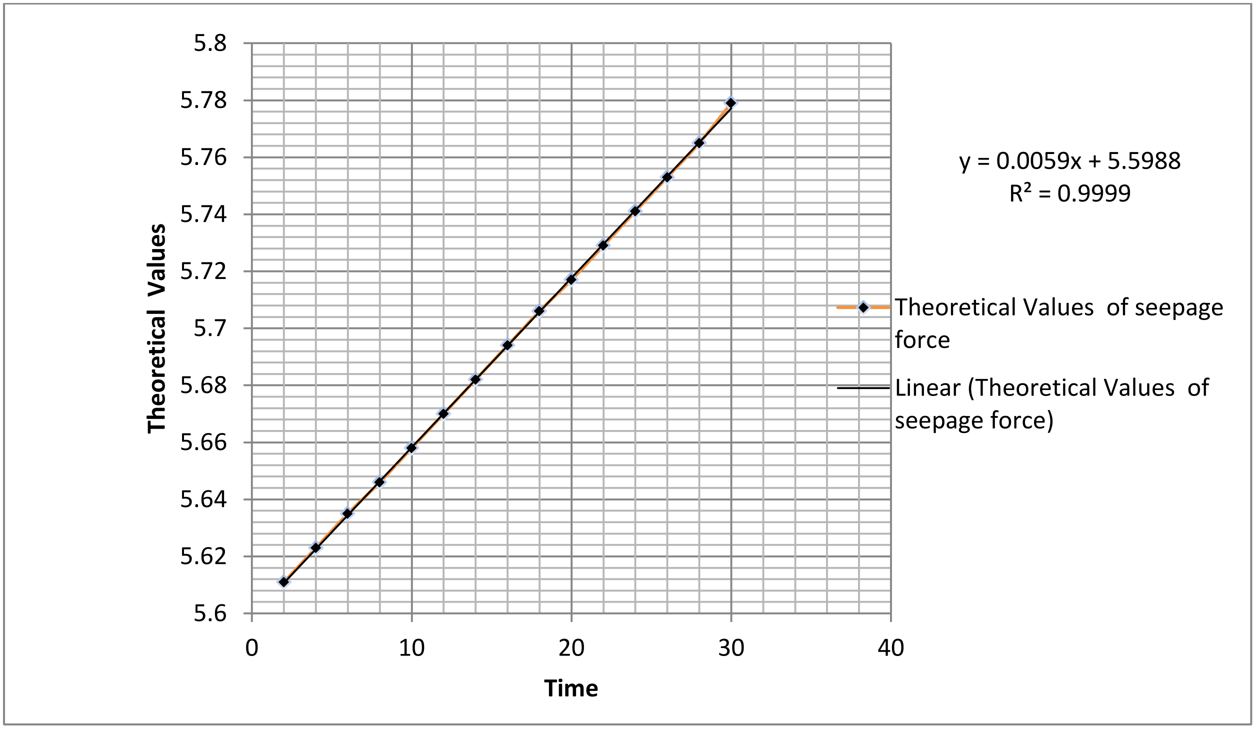

Figure 4: Theoretical values of seepage force at different Time

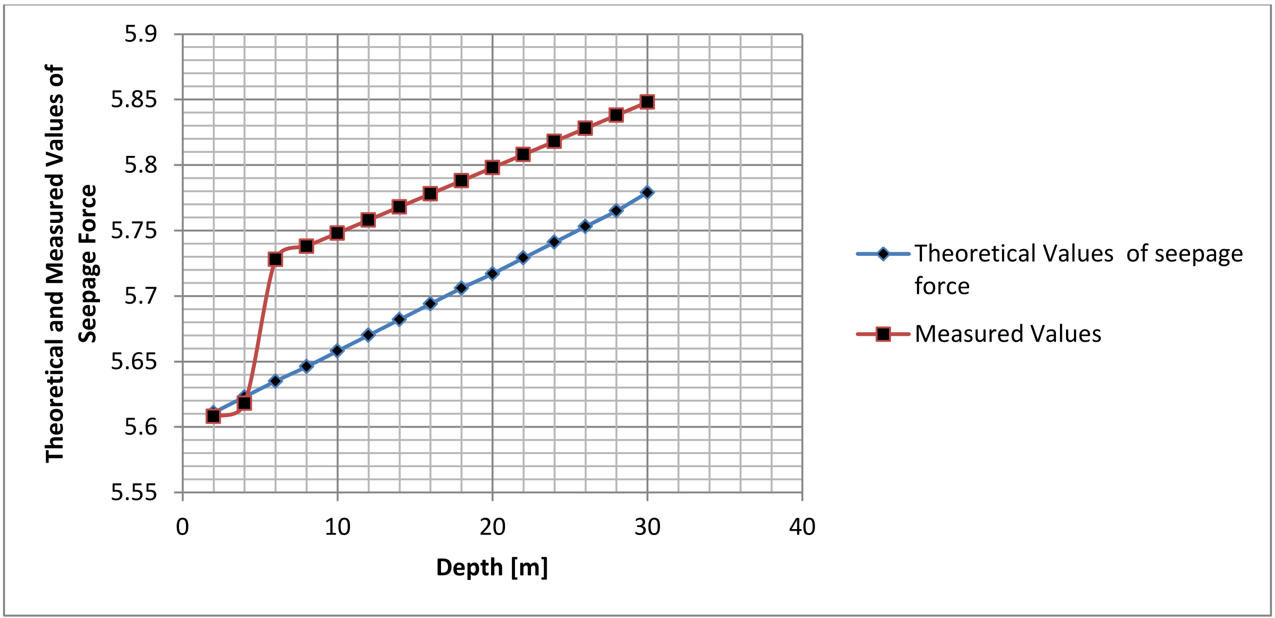

Figure 5: Comparison of Theoretical and Measured Values of Seepage Force at Different Depth

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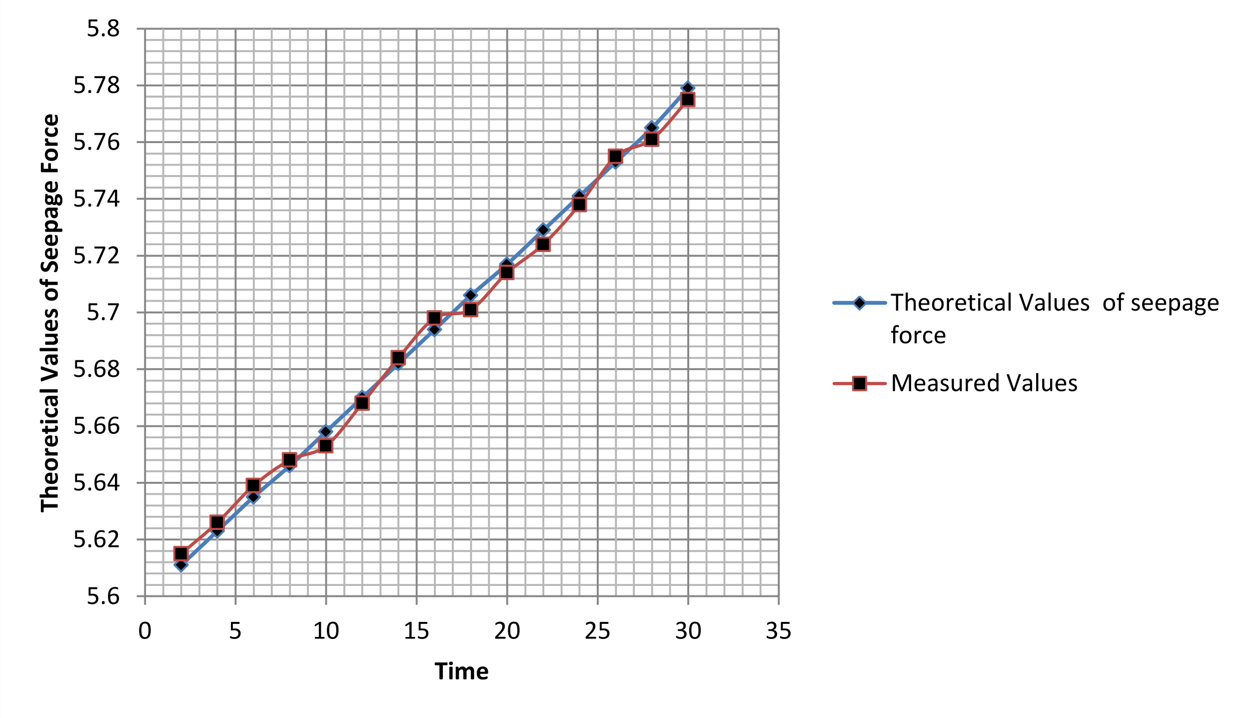

Figure 6: Comparison of Theoretical and Measured Values of Seepage Force at Different Time

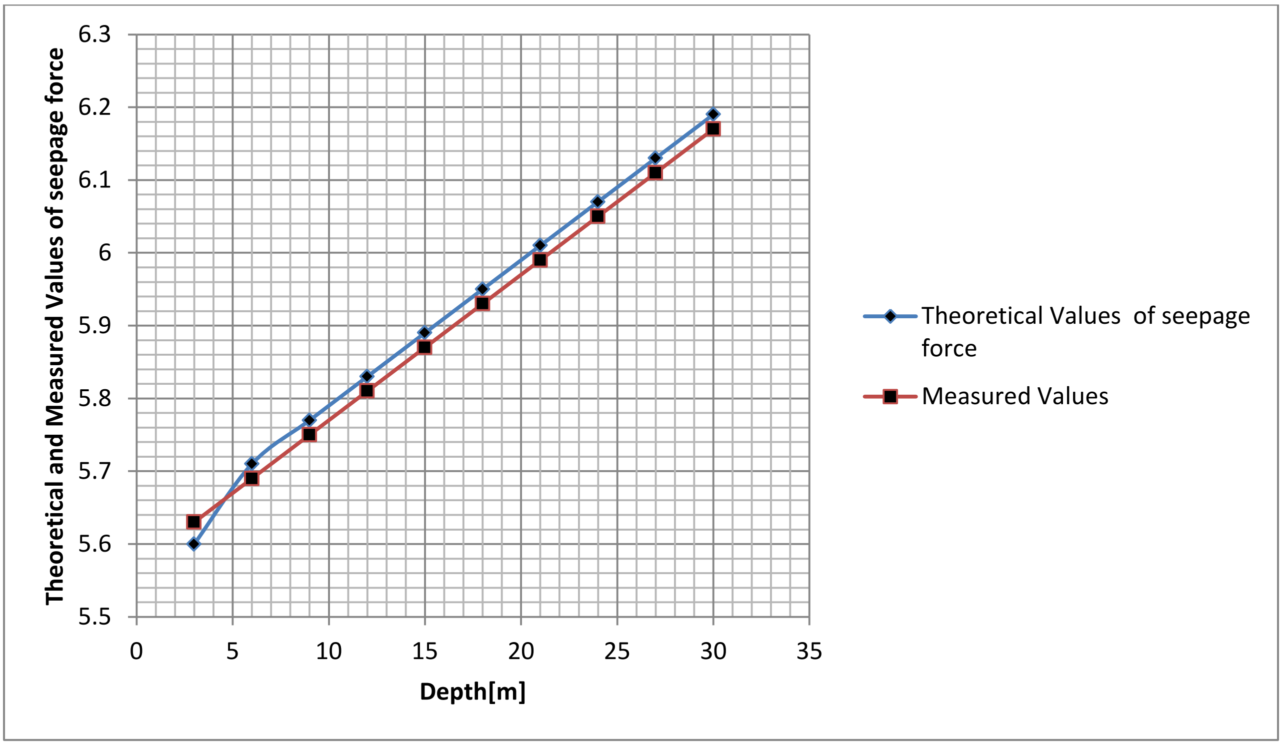

Figure 7: Comparison of Theoretical and Measured Values of Seepage Force at Different Depth

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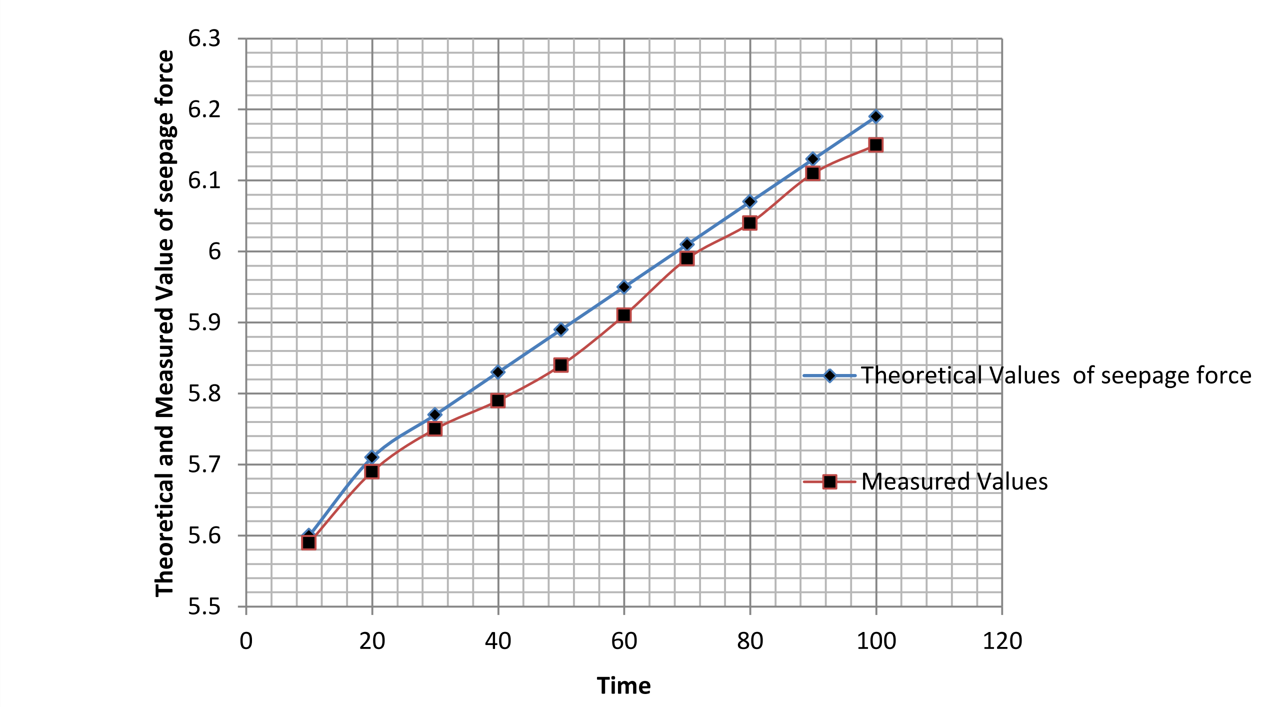

Figure 8: Comparison of Theoretical and Measured Values of Seepage Force at Different Time

The figures I -8 presented shows that that the deposition of the formation express rapid rate of seepage force in the study location, such condition are from such influences of the soil structural deposition in the study location, the expressions developed rapid seepage forces in the formation, this is through the level of the fluid, it is confirm to depended on the depositional setting of the formation , the expression of linear deposition where found in the figures, the depositional forces varies under the influences of structural strata setting of the formation. The theoretical values experienced linear deposition while the measured values maintained the same condition as it is expressed in the figures presented above. The expressed model developing linear seepage forces definitely influences geotechnical properties of the soil as it is reflected in the soil bearing pressures, establishment of such types of seepage forces definitely influences the characteristics fluid flow dynamics between various intercedes of the strata, the theoretical and the measured value established fitness showing validation of the model.

5. Conclusion

The rate of seepage forces are caused by the rate of structural setting of the formation this are reflected when water flow through a soil, the water head is dissipated in viscous friction during energy dissipation , the drag forces is exerted on the soil particles in the direction of flow it is confirmed that when water is in soil column, the fluid height in the surface are raised whereby the water pressured at the bottom of the soil sample increased and the drag forces of the soil particles become greater, the height of the buoyant weight of the particles are found to be in a balanced state at critical height, the expressed governing equation were developed under the influences of these condition as it stated to generate seepage forces in a formation, it is developed through these condition from the material balances that produces the governing equation, the derived solution generated the model that expressed the seepage forces through model simulation, it generated theoretical values expresses the seepage force with slight variation under the influences of homogeneous structural setting of the formation, these include the geophysical properties of the soil formation under flow of fluid between the intercedes of the strata.

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